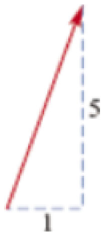
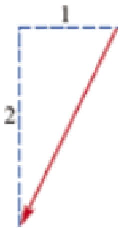
Exercise 17A: Solutions

1 **a** $\begin{bmatrix}1\\5\end{bmatrix}$ is the vector " 1 across to the right and 5 up."

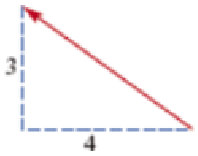


b $\begin{bmatrix}0\\-2\end{bmatrix}$ is the vector " 2 down."

c $\begin{bmatrix}-1\\-2\end{bmatrix}$ is the vector " 1 across to the left and 2 down."



d $\begin{bmatrix}-4\\3\end{bmatrix}$ is the vector " 4 across to the left and 3 up."



2 $\boldsymbol{u} = \begin{bmatrix}6-1\\6-5\end{bmatrix} = \begin{bmatrix}5\\1\end{bmatrix}$

$a = 5, b = 1$

3 $\boldsymbol{v} = \begin{bmatrix}2--1\\-10-5\end{bmatrix} = \begin{bmatrix}-3\\15\end{bmatrix}$

$a = 3, b = -15$

4 **a** $\overrightarrow{OA} = \begin{bmatrix}1-0\\-2-0\end{bmatrix} = \begin{bmatrix}1\\-2\end{bmatrix}$

b $\overrightarrow{AB} = \begin{bmatrix}3-1\\0--2\end{bmatrix} = \begin{bmatrix}2\\2\end{bmatrix}$

c $\overrightarrow{BC} = \begin{bmatrix}2-3\\-3-0\end{bmatrix} = \begin{bmatrix}-1\\-3\end{bmatrix}$

d $\overrightarrow{CO} = -\overrightarrow{OC} = \begin{bmatrix}-2\\3\end{bmatrix}$

e $\overrightarrow{CB} = -\overrightarrow{BC} = \begin{bmatrix}1\\3\end{bmatrix}$

5 a i $\a + \b = \begin{bmatrix} 1 \\ 2 \end{bmatrix} + \begin{bmatrix} 1 \\ -3 \end{bmatrix}$

$= \begin{bmatrix} 1+1 \\ 2+-3 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$

ii $2\c - \a = 2 \times \begin{bmatrix} -2 \\ 1 \end{bmatrix} - \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

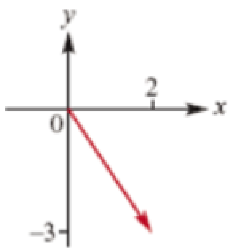
$= \begin{bmatrix} -4-1 \\ 2-2 \end{bmatrix} = \begin{bmatrix} -5 \\ 0 \end{bmatrix}$

iii $\a + \b - \c = \begin{bmatrix} 2 \\ -1 \end{bmatrix} - \begin{bmatrix} -2 \\ 1 \end{bmatrix}$

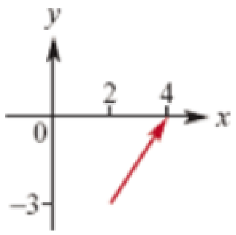
$= \begin{bmatrix} 2--2 \\ -1-1 \end{bmatrix} = \begin{bmatrix} 4 \\ -2 \end{bmatrix}$

b $\a + \b = \begin{bmatrix} 2 \\ -1 \end{bmatrix} = -\c \therefore \a + \b$ is parallel to $\c$.

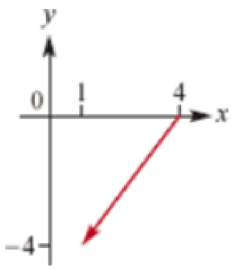
6 a



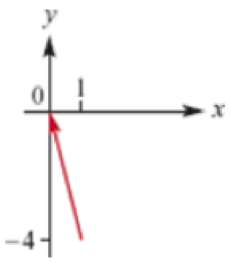
b



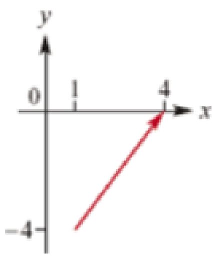
c



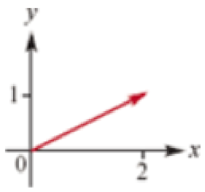
d



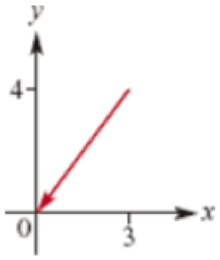
e



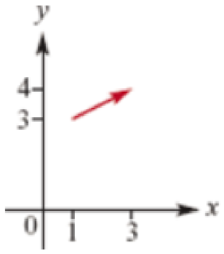
7 a



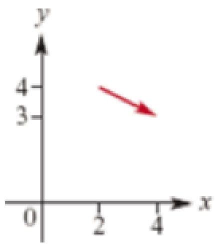
b



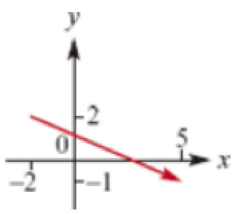
c



d



e



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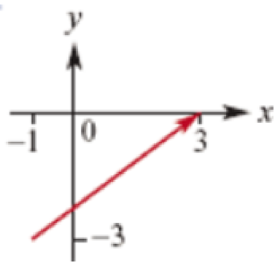
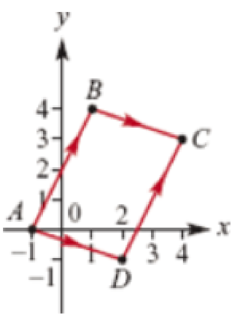


8 From the graphs above it can be seen that $\mathbf{a}$ and $\mathbf{c}$ are parallel.

9 **a & b**



c i $\overrightarrow{AB} = \begin{bmatrix} 1 - -1 \\ 4 - 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$

$\overrightarrow{DC} = \begin{bmatrix} 4 - 2 \\ 3 - -1 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$

$\therefore \overrightarrow{AB} = \overrightarrow{DC}$

ii $\overrightarrow{BC} = \begin{bmatrix} 4 - -1 \\ 3 - 4 \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$

$\overrightarrow{AD} = \begin{bmatrix} 2 - -1 \\ -1 - 0 \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$

$\therefore \overrightarrow{BC} = \overrightarrow{AD}$

d $ABCD$ is a parallelogram.

10 $m\begin{bmatrix} 3 \\ -3 \end{bmatrix} + n\begin{bmatrix} 2 \\ 4 \end{bmatrix} = \begin{bmatrix} 3m \\ -3m \end{bmatrix} + \begin{bmatrix} 2n \\ 4n \end{bmatrix}$

$= \begin{bmatrix} 3m & +2n \\ -3m & +4n \end{bmatrix} = \begin{bmatrix} -19 \\ 61 \end{bmatrix}$

$3m + 2n = -19$

$6m + 4n = -38$ ①

$-3m + 4n = 61$ ②

① − ②:

$9m = -99$

$m = -11$

$-33 + 2n = -19$

$2n = -19 + 33$

$= 14$

$n = 7$

11a i $\overrightarrow{MD} = \overrightarrow{MA} + \overrightarrow{AD}$

$= \frac{1}{2}\overrightarrow{BA} + \boldsymbol{b}$

$= -\frac{1}{2}\overrightarrow{AB} + \boldsymbol{b}$

$= \boldsymbol{b} - \frac{1}{2}\boldsymbol{a}$

ii $\overrightarrow{MN} = \overrightarrow{MA} + \overrightarrow{AD} + \overrightarrow{DN}$

$= \frac{1}{2}\overrightarrow{BA} + \boldsymbol{b} + \frac{1}{2}\overrightarrow{DN}$

$= -\frac{1}{2}\overrightarrow{AB} + \boldsymbol{b} + \frac{1}{2}\overrightarrow{DC}$

$= -\frac{1}{2}\boldsymbol{a} + \boldsymbol{b} + \frac{1}{2}\boldsymbol{a}$

= \b

b $\overrightarrow{MN} = \overrightarrow{AD}$

(both are equal to $\boldsymbol{b}$)

12a $\vec{CB} = \vec{CA} + \vec{AB}$

$= -\backslash\mathrm{b} + \backslash\mathrm{a} = \backslash\mathrm{a} - \backslash\mathrm{b}$

$\vec{MN} = \vec{MA} + \vec{AN}$

$= -\frac{1}{2}\backslash\mathrm{a} + \frac{1}{2}\backslash\mathrm{b}$

$= \frac{1}{2}(\backslash\mathrm{b} - \backslash\mathrm{a})$

b $\vec{MN}$ is half the length of $\vec{CB}$, is parallel to $\vec{CB}$ and in the opposite direction to $\vec{CB}$.

13a $\vec{CD} = \vec{AF} = \backslash\mathrm{a}$

b $\vec{ED} = \vec{AB} = \backslash\mathrm{b}$

c The regular hexagon can be divided into equilateral triangles, showing that $\vec{BE} = 2\vec{AF} = 2\backslash\mathrm{a}$.

d Likewise, $\vec{FC} = 2\vec{AB} = 2\backslash\mathrm{b}$

e $\vec{FA} = -\vec{AF} = -\backslash\mathrm{a}$

f $\vec{FB} = \vec{FA} + \vec{AB}$

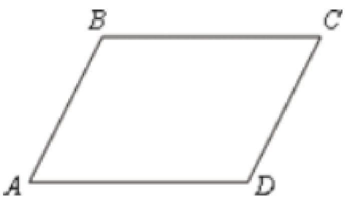
$= -\backslash\mathrm{a} + \backslash\mathrm{b} = \backslash\mathrm{b} - \backslash\mathrm{a}$

g $\vec{FE} = \vec{FA} + \vec{AB} + \vec{BE}$

$= -\backslash\mathrm{a} + \backslash\mathrm{b} + 2\backslash\mathrm{a}$

$= \backslash\mathrm{a} + \backslash\mathrm{b}$

14



a $\vec{DC} = \vec{AB} = \backslash\mathrm{a}$

b $\vec{DA} = -\vec{BC} = -\backslash\mathrm{b}$

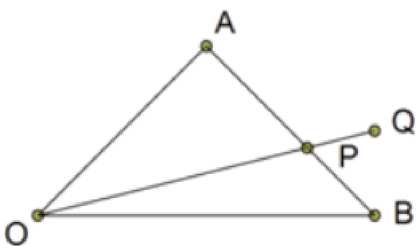
c $\vec{AC} = \vec{AB} + \vec{BC} = \backslash\mathrm{a} + \backslash\mathrm{b}$

d $\vec{CA} = -\vec{AC} = -\backslash\mathrm{a} - \backslash\mathrm{b}$

e $\vec{BD} = \vec{BA} + \vec{AD}$

$= -\backslash\mathrm{a} + \backslash\mathrm{b} = \backslash\mathrm{b} - \backslash\mathrm{a}$

15



a $\overrightarrow{BA} = \overrightarrow{BO} + \overrightarrow{OA} = \backslash a - \backslash b$

b $\overrightarrow{AB} = -\overrightarrow{BA} = \backslash b - \backslash a$

$\overrightarrow{PB} = \frac{1}{3}\overrightarrow{AB} = \frac{1}{3}(\backslash b - \backslash a)$

c $\overrightarrow{AP} = \frac{2}{3}\overrightarrow{AB} = \frac{2}{3}(\backslash b - \backslash a)$

$$\begin{aligned}\overrightarrow{OP} &= \overrightarrow{OA} + \overrightarrow{AP} \\ &= \backslash a + \frac{2}{3}(\backslash b - \backslash a) \\ &= \frac{1}{3}\backslash a + \frac{2}{3}\backslash b \\ &= \frac{1}{3}(\backslash a + 2\backslash b)\end{aligned}$$

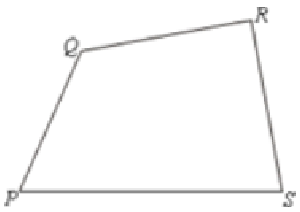
d

$$\begin{aligned}\overrightarrow{PQ} &= \frac{1}{3}\overrightarrow{OP} \\ &= \frac{1}{3} \times \frac{1}{3}(\backslash a + 2\backslash b) \\ &= \frac{1}{9}(\backslash a + 2\backslash b)\end{aligned}$$

e $\overrightarrow{BP} = -\overrightarrow{PB} = \frac{1}{3}(\backslash a - \backslash b)$

$$\begin{aligned}\overrightarrow{BQ} &= \overrightarrow{BP} + \overrightarrow{PQ} \\ &= \frac{1}{3}(\backslash a - \backslash b) + \frac{1}{9}(\backslash a + 2\backslash b) \\ &= \frac{1}{9}(4\backslash a - \backslash b)\end{aligned}$$

16



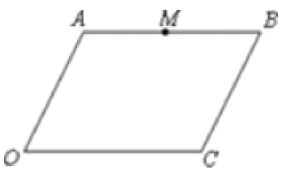
a $\overrightarrow{PR} = \overrightarrow{PQ} + \overrightarrow{QR} = \backslash u + \backslash v$

b $\overrightarrow{QS} = \overrightarrow{QR} + \overrightarrow{RS} = \backslash v + \backslash w$

c

$$\begin{aligned}\overrightarrow{PS} &= \overrightarrow{PQ} + \overrightarrow{QR} + \overrightarrow{RS} \\ &= \backslash u + \backslash v + \backslash w\end{aligned}$$

17



a $\overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{AB} = \backslash u + \backslash v$

$$\vec{AM} = \vec{MB}$$
$$= \frac{1}{2}\vec{AB} = \frac{1}{2}\backslash\mathbf{v}$$
$$\vec{OM} = \vec{OA} + \vec{AM}$$
$$= \backslash\mathbf{u} + \frac{1}{2}\backslash\mathbf{v}$$

b $$\vec{CM} = \vec{CB} + \vec{BM}$$
$$= \backslash\mathbf{u} + \frac{1}{2}\vec{BA}$$
$$= \backslash\mathbf{u} - \frac{1}{2}\backslash\mathbf{v}$$

c $$\vec{CP} = \frac{2}{3}\vec{CM}$$
$$= \frac{2}{3}\left(\backslash\mathbf{u} - \frac{1}{2}\backslash\mathbf{v}\right)$$
$$= \frac{2}{3}\backslash\mathbf{u} - \frac{1}{3}\backslash\mathbf{v}$$

d $$\vec{OP} = \vec{OC} + \vec{CP}$$
$$= \backslash\mathbf{v} + \left(\frac{2}{3}\backslash\mathbf{u} - \frac{1}{3}\backslash\mathbf{v}\right)$$
$$= \frac{2}{3}\backslash\mathbf{u} + \frac{2}{3}\backslash\mathbf{v}$$
$$= \frac{2}{3}(\backslash\mathbf{u} + \backslash\mathbf{v}) = \frac{2}{3}\vec{OB}$$

Since OP is parallel to OB and they share a common point O, OP must be on the line OB. Hence P is on $\vec{OB}$

e Using the result from part **d**,
$OP : PB = 2 : 1$.